Assessing the State of Population Health by Age-adjusted Life Expectancies

U. Feldmann, R. F. Mehnert
Institute of Medical Biometry, Epidemiology and Medical Informatics, University of Saarland, Germany

Summary

Objectives: A gain in life expectancy of a population is commonly interpreted as an effect of improved health care. After the reunion of Germany in 1990 life expectancy at birth grew extremely in the new federal states. Within one decade after reunion the new federal states had a gain in life expectancy of about five years while the gain in the old federal states was only about two years.

Methods: It has been widely argued that this phenomenon is caused by an obviously improved public health service and environmental protection or even by an increased social status in the new federal states. On the other hand, the median population age grew rapidly in the new federal states, caused by a dramatic reduction of the birth rate as well as a high emigration rate of young people. Using real time series for three selected federal states and for the total Federal Republic of Germany, it is derived that most of the gain in life expectancy is explained by population ageing.

Results: An elementary probabilistic procedure is proposed allowing for estimating the amount in life expectancy not attributable to population ageing.

Conclusions: The age-adjusted life expectancy can be regarded as an unbiased measure of a populations' state of health that stays comparable both over time and across countries.

Keywords

Age at death, Bayes' theorem, fertility, health care, life expectancy, life tables, migration, population ageing, state of health of populations

Methods Inf Med 2006; 45: 275–80

1. Introduction

Life table analysis owns an old tradition. In 1691 the astronomer and mathematician Edmond Halley [1] was one of the first who founded mathematical principles of birth-cohort life tables. In 1958 these principles were transferred to the estimation of survival curves in medicine by Cutler and Ederer [2]. In particular, they adjusted cumulative survival rates for censored data.

In the context of birth-cohort life table analysis censoring occurs in the face of migration in a population. Formulas for adjusting migration have been known for several centuries. Until now no attempt seems to be having made to estimate the consequences of decreasing birth rates in a population. Not an improved public health service but birth rate reduction is the main reason why the median age of populations increases and therefore causes an over-estimation of life expectancy.

In 2005 the magazine Nature brought public attention to this situation. Sanderson and Scherbov [3] stated that average remaining lifetimes can increase as human populations age. In order to take this effect into account they proposed the concept of median age of a population standardized for expected remaining years of life.

The present paper is motivated by the question why the new federal states of Germany had such an extreme gain in life expectancy after reunion. A novel approach to birth-cohort life table analysis is proposed in section 2. Bayes' theorem of probability is applied to model age-specific mortality rates in terms of the age distribution at death, while the population age distribution is standardized. This allows for adjusting life expectancy for population ageing.

In section 3 time series of demographic data published in a medical thesis [4] are reanalyzed. The conclusion of this thesis was that gain in life expectancy is associated with an increased median population age. The present paper transfers this heuristic finding to a mathematical property, proven by elementary probabilistic arguments.

2. Adjusting Life Expectancy for Population Ageing

In this section we briefly repeat basic concepts of birth-cohort life table analysis and propose an approach adjusting life expectancy for changes in the age structure of a population. This seems to become necessary, since population ageing is a most doubtful measure of improved health care or environmental protection, but is mainly caused by decreasing birth rates.

A life table exclusively depends on age-specific mortality rates. Age-specific mortality rates relate the number of individuals $N(x)$ who entered a certain calendar year at age x and the number of the subset of these individuals $D(x)$ who died during this calendar year,

$$q(x) = \frac{D(x)}{N(x)}. \tag{1}$$

The crude mortality rate, that means the total fraction of individuals dying during the calendar year, is expressed as

$$q = \frac{D}{N} \text{ where } D = \sum_x D(x) \text{ and } N = \sum_x N(x). \tag{2}$$

The cumulative survival rate until age x is computed by

$$S(x) = \prod_{X=0}^{X=x-1} \left(1 - q(X)\right) \text{ and } S(0) = 1. \quad (3)$$

This determines the fraction of newborns at the beginning of the calendar year who will survive at least x years and is referred to as a birth-cohort life table. Life table analysis establishes a forecasting up to x years proceeding from a certain calendar year and using age-specific mortality rates obtained in this calendar year. In order to stabilize forecasting, data from adjacent calendar years may be included in the computation of the cumulative survival rate, referred to as a period life table.

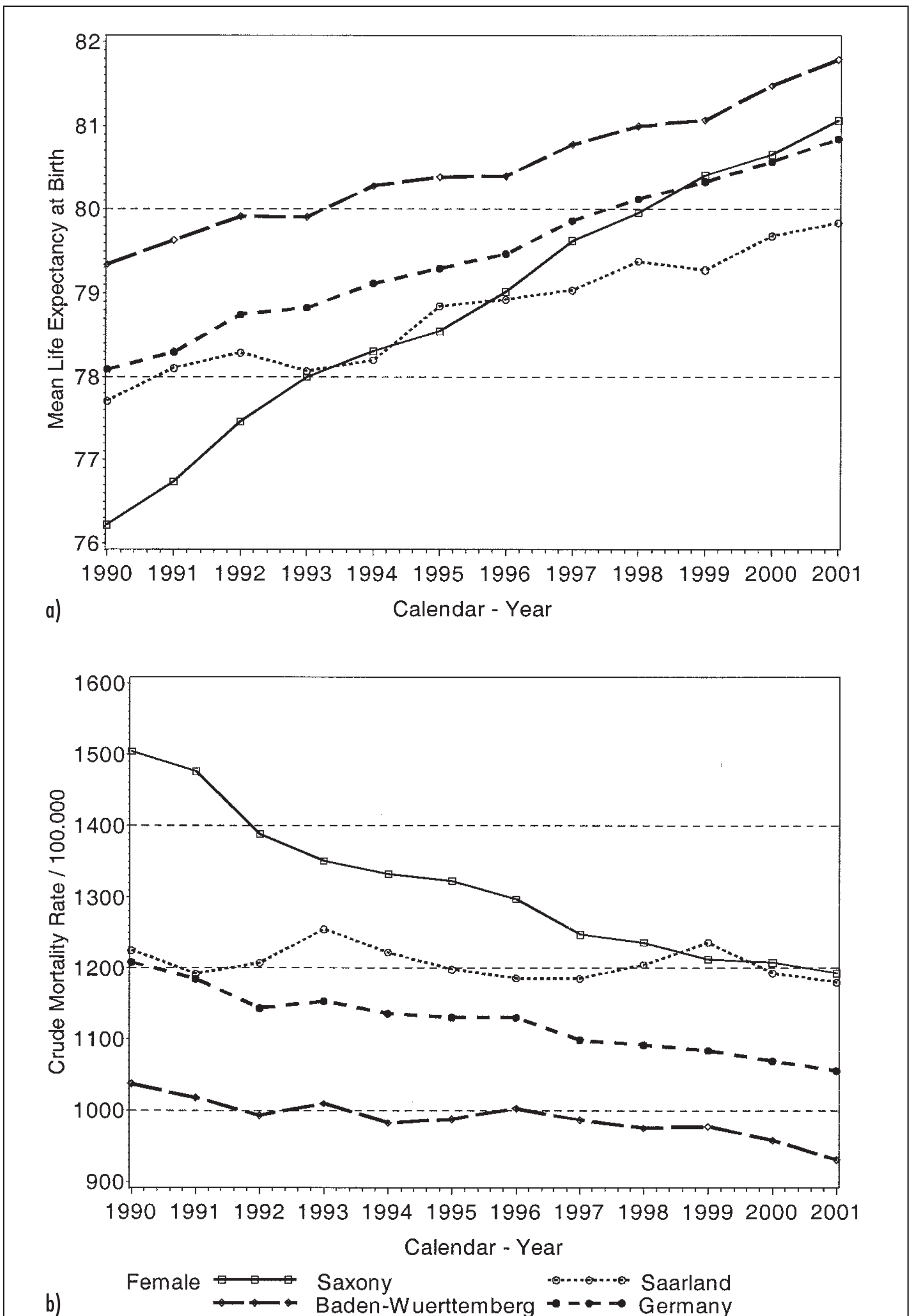

Fig. 1 Measures of populations' state of health: a) mean life expectancy at birth, b) crude mortality rate

The median life expectancy at birth is defined by

$$S(\hat{x}) = \tfrac{1}{2}.$$

The remaining median life expectancy of an individual already being at age x at the beginning of the calendar year is computed as

$$S(\hat{x} + x) = \tfrac{1}{2} \cdot S(x). \quad (4)$$

Median life expectancy $\hat{x}$ means the remaining life time that 50% of all individuals being at the beginning of the calendar year at age x will reach or exceed.

In life table analysis the effect of migration in a population, in particular the effect of emigration, is modelled by the concept of censored data. Suppose that $W(x)$ is the number of individuals at age x who are withdrawn during the calendar year. Then the age-specific mortality rates are commonly adjusted by

$$q(x) = \frac{D(x)}{N(x) - \tfrac{1}{2} \cdot W(x)}. \quad (5)$$

If emigration concerns young people only, there will be no remarkable effect of this adjustment with respect to life expectancy. This is because mortality rates of young people are near to zero while mortality rates of elderly individuals are hardly effected by the formula (5).

An approach to life table analysis is required that takes alterations in the age structure of the population into account.

Consider the fraction $n(x) = pr(x)$ of individuals at age x among all individuals being alive at the beginning of the calendar year and the fraction $d(x) = pr(x|D)$ of individuals who died at age x among all individuals who died during this calendar year. The total fraction of deaths is $q = pr(D)$.

The above fractions are formulated in terms of probabilities. Using Bayes' theorem of probability, the age-specific mortality rates are written as

$$q(x) = q \cdot \frac{d(x)}{n(x)}. \quad (6)$$

The numerical results of these rates are just the same as in formula (1), since

$$n(x) = \frac{N(x)}{N} \text{ and } d(x) = \frac{D(x)}{D}. \quad (7)$$

But the equation (6) allows for the mathematical modelling of population ageing. Now the age-specific mortality rates depend on the age-density distributions of those people who entered the calendar year and the subset of those people who died during the calendar year.

Age-density distribution means

$$\sum_x n(x) = 1 \text{ and } \sum_x d(x) = 1. \quad (8)$$

A reduction of the birth rate or an emigration of young people hardly effects the age distribution at death but certainly alters the population age distribution. This implies that $d(x)$ would be left hardly effected in this instance while $n(x)$ increases for elderly people. If the crude mortality rate is assumed to be fixed, the age-specific mortality rates (6) for elderly people will decrease. Hence, life expectancy may increase simply because of the process of demographic ageing of a population.

In order to adjust this effect it is obvious to replace the observed population age-density distribution $n(x)$ by a standard age-density distribution $n^*(x)$, for instance derived from the WHO world standard population or the European standard population at 1990.

Even the crude mortality rate is influenced by population ageing. Bayes' theorem of total probability leads to the identity

$$q = pr(X<x) \cdot pr(D|X<x) + pr(X \geq x) \cdot pr(D|X \geq x). \quad (9)$$

Here $pr(X<x)$ means the fraction of people being younger than x years at the beginning of the calendar year, while $pr(D|X<x)$ is the fraction of individuals who died during the calendar year at an age younger than x years.

For young people the mortality rate is negligible. An emigration of young people or a birth rate reduction increases the fraction of elderly people $pr(X \geq x)$ but leaves their mortality rate $pr(D|X \geq x)$ hardly effected. Hence emigration of young people or birth rate reduction leads to an increase of the crude mortality rate.

In order to adjust the crude mortality rate for population ageing the fractions of younger and elderly people are derived from the standard population. Let x^* be the median age of the standard population, then the median age-adjusted mortality rate reads

$$q^* = \tfrac{1}{2} \cdot pr\left(D \middle| X < x^*\right) + \tfrac{1}{2} \cdot pr\left(D \middle| X \geq x^*\right). \quad (10)$$

This is computed by

$$q^* = \frac{q_0 + q_1}{2} \text{ with } q_0 = \frac{D_0}{N_0} \text{ and } q_1 = \frac{D_1}{N_1}. \quad (11)$$

N_0 and D_0 are the numbers of living and died individuals who are younger than x^* years, while N_1 and D_1 are the respective numbers in the complementary population of the elderly.

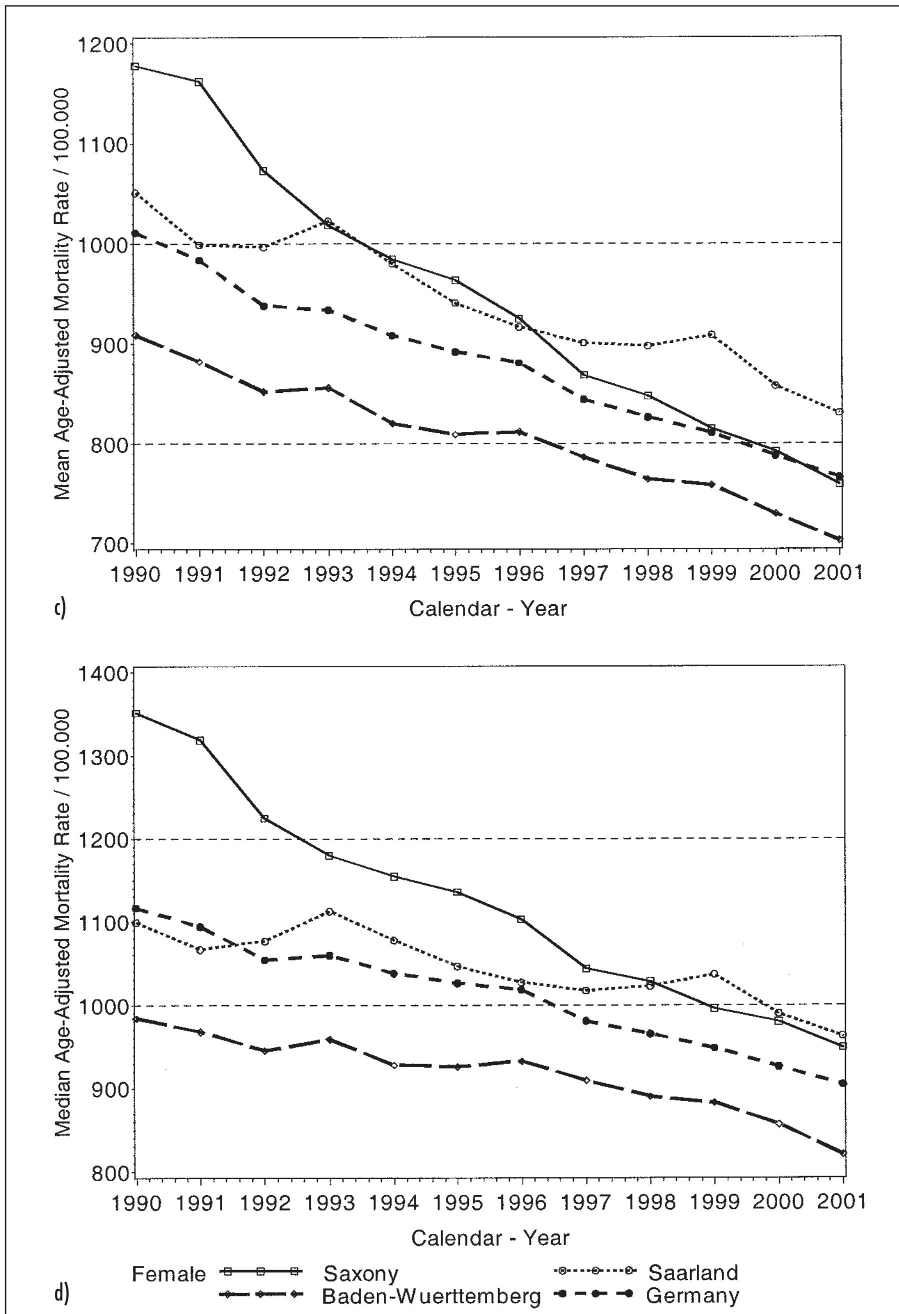

Fig. 1 Continued Measures of populations' state of health: c) average age-standardized mortality rate, d) median age-standardized mortality rate

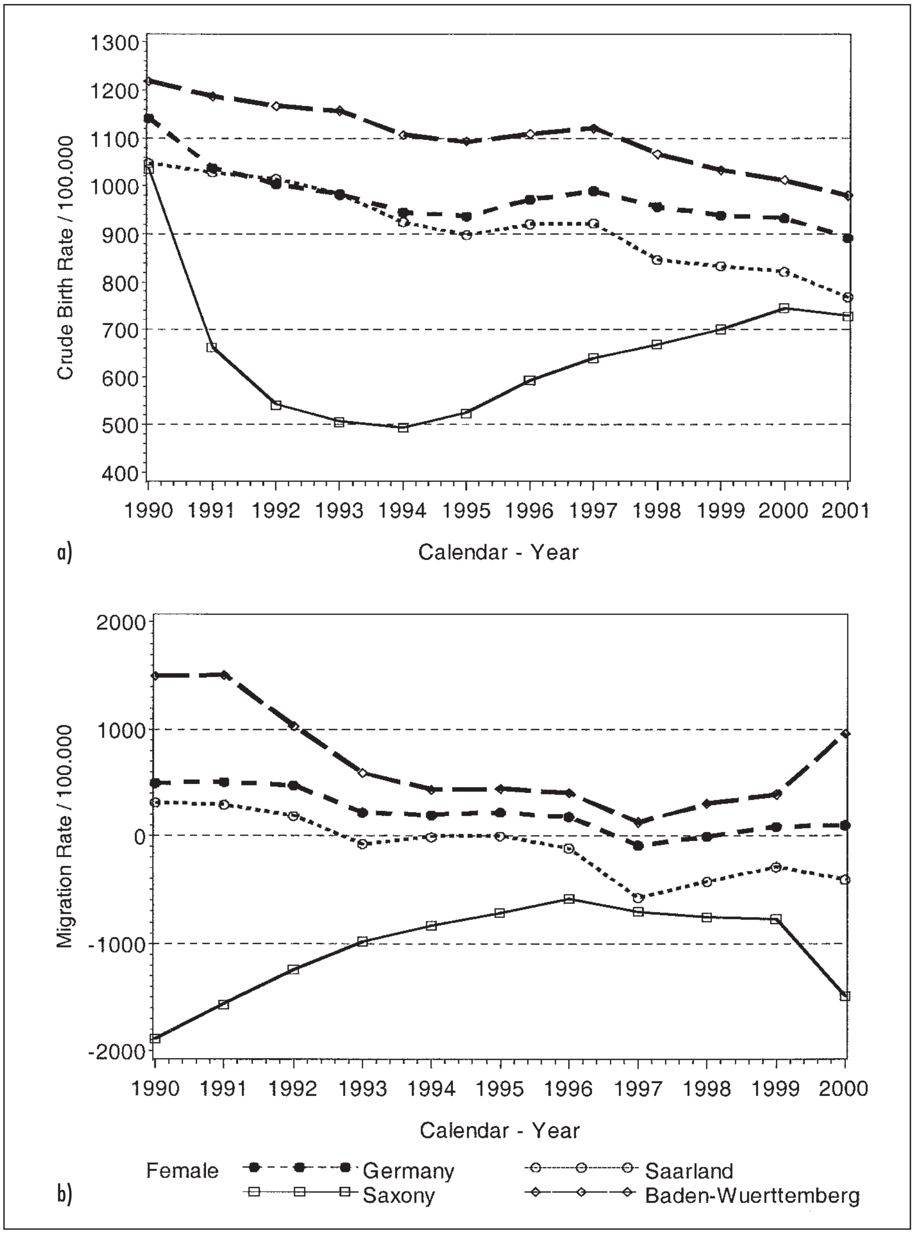

Fig. 2 Reasons for population ageing: a) crude birth rate, b) migration rate

The age-specific mortality rates adjusted for population ageing read

$$q^*(x) = q^* \cdot \frac{d(x)}{n^*(x)}. \qquad (12)$$

Then the age-adjusted life table as well as the respective life expectancy depend on the age-density distribution at death, while the population age-density distribution is completely standardized. This property adjusts life expectancy for population ageing and makes it comparable over calendar years as well as across countries.

3. Applications

The applications are based on time series of demographic data observed in the decade after the reunion of the Federal Republic of Germany [4]. In the following all considerations are restricted to female citizens.

The comparison includes the federal states of Saxony (2.533 Mio –11%), Saarland (0.554 Mio –1%), Baden-Wuerttemberg (4.995 Mio +8%) and the total Federal Republic of Germany (41.088 Mio +3%). In the brackets the female population size is indicated for the calendar year 1990 as well as its change until 2001. Saxony was chosen to represent the new federal states, Baden-Wuerttemberg was chosen as the old federal state having the highest life expectancy and Saarland is one of the old federal states owning worse life expectancy.

The progress of surrogate parameters usually indicating the state of health of populations is demonstrated in Figure 1. While the old federal states show a nearly parallel progress of mean life expectancy at birth, in Saxony a rapid increase is to be recognized (Fig. 1a). This is accompanied by a steep decreasing crude mortality rate (Fig. 1b).

The progress of the familiar direct age-standardized mortality rate is shown in Figure 1c. This is computed as the weighted average of the observed age-specific mortality rates

$$q^* = \sum_x n^*(x) \cdot q(x), \qquad (13)$$

weighted by the age-density distribution of the European female standard population at 1990. It must be emphasized that this mortality rate still depends on the population age-density distribution. The median age-standardized mortality rate proposed in this paper (Fig. 1d), does not depend on the population age-density distribution and is almost greater than the average age-standardized rate but of course less than the crude mortality rate.

The regress of birth rates is demonstrated in Figure 2a. While Baden-Wuerttemberg shows the lowest reduction of birth rates, in Saxony a dramatic regress of birth rates must be stated. Within only three years birth rates were reduced to more than 50%.

The reduction of birth rates is accompanied by a remarkable emigration from Saxony (see Fig. 2b). In 1990 about 2% of all women emigrated from Saxony. This concerned mostly young women. About 8% of the women aged between 20 and 25 years emigrated during this year. After one decade the female population size was reduced by 11%.

While there is a strong increase in life expectancy in Saxony (Fig. 1a), the birth rates fall dramatically and emigration of young women continues on a high level (Fig. 2). In order to interpret this behavior one first has to consider the median of the age distribution at death, since an effect of improved health care should be reflected by an increased median age at death.

As seen in Figure 3a, the progress of the median age at death in Saxony approximately corresponds to that of total Germany. Hence the strong increase of the life expectancy in Saxony (Fig. 1a) is not reflected by the median age at death (Fig. 3a). Even a decrease of the median age at death is observed in all populations since the years 1998/1999.

On the other hand the median population age grows over time (Fig. 3b). But in Saxony an increase of more than four years is observed while the other populations only have an increase of about two years. The rapid population ageing in Saxony is explained by the dramatic reduction of the birth rates as well as the remarkable emigration of young citizens (Fig. 2).

The progress of the median life expectancies at birth is demonstrated in Figure 4a. It must be stated that the median life expectancies are about three years higher than the corresponding mean life expectancies in Figure 1a. However, also the median life expectancies confirm the rapid increase of lifetime in Saxony. For the calendar year 2001 the mean life expectancy as well as the median life expectancy in Saxony even exceed the corresponding life expectancies of total Germany.

After population age adjustment as outlined in section 2 the progress of median life expectancies alters essentially (see Fig. 4b). The age-adjusted median life expectancies in Saxony show a nearly parallel progress compared with total Germany. But the adjusted lifetimes in Saxony are always lower than in total Germany. Since the calendar years 1998/1999, for Saxony, Saarland and total Germany the age-adjusted median life expectancies even fall. This is in agreement with the falling median ages at death (Fig. 3a). Only in Baden-Wuerttemberg the age-adjusted median lifetime increases further.

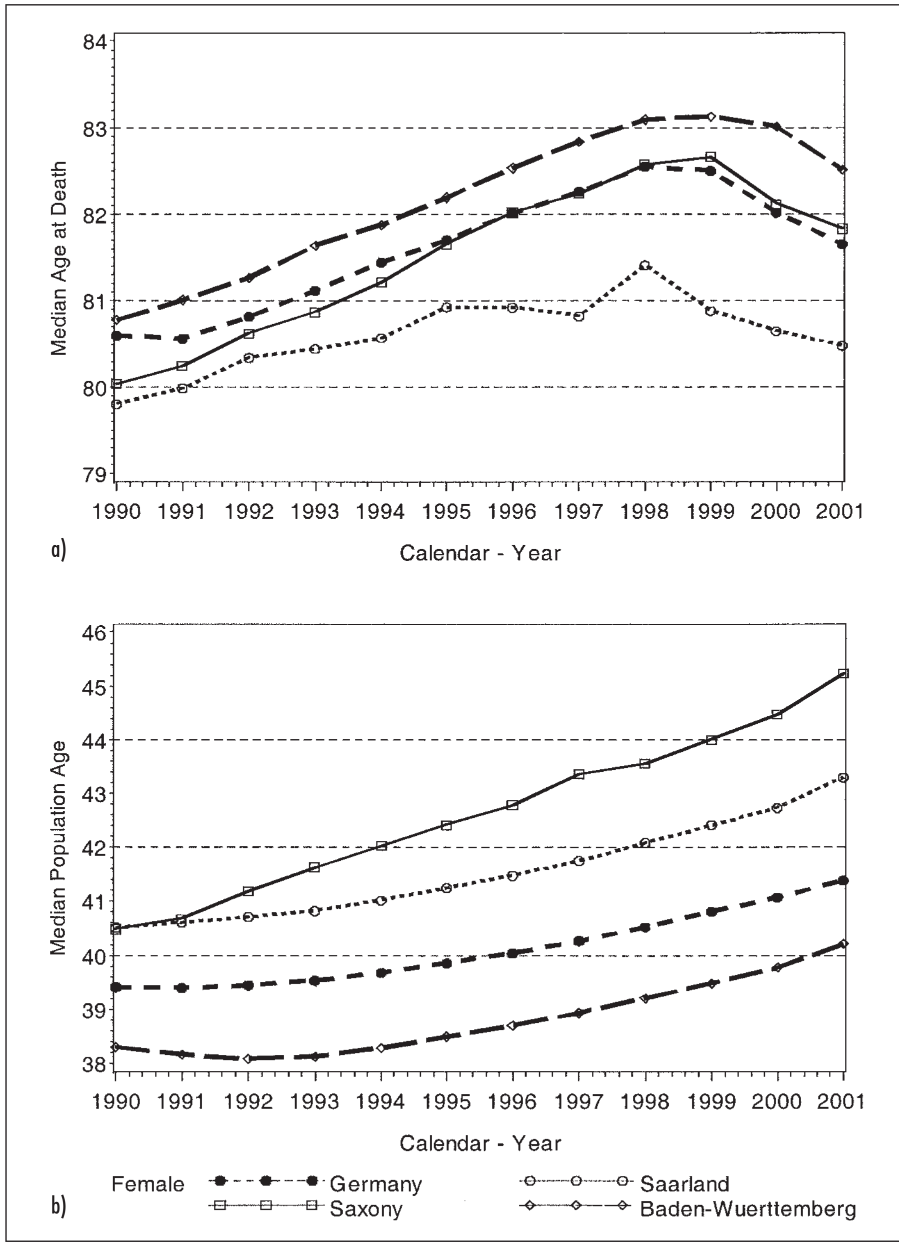

Fig. 3 Measures of ageing: a) median age at death, b) median population age

4. Discussion

Life expectancy depends on the age-specific mortality rates of a population. But the age-specific mortality rates themselves depend on the age distributions of those people who enter the calendar year as well as of those who die during the calendar year. The first is the population age distribution and the latter is the age distribution at death.

The basic idea of this paper is that the median of the age distribution at death can be regarded as a measure for the quality of a health care system. On the other hand, the median of the population age distribution is a worse indicator for the state of health of populations, since it is heavily confounded by demographic factors like fertility and migration.

The innovation of this paper is to remove the effect of population ageing completely from the computation of age-specific mortality rates. After standardization the age-specific mortality rates depend on the age distribution at death, while the population age distribution is replaced by a standard population. This allows for an age-adjusted computation of the life expectancy, which can be regarded as an unconfounded measure of the state of health of populations.

Sanderson and Scherbov [3] proposed to use the remaining lifetime adjusted for the median population age as a measure for population ageing. Their approach for the first time explains the effect of falling remaining life expectancies. Their finding is that median ages can increase so rapidly relative to improvements in mortality that remaining life expectancies fall.

In the present paper population age adjustment is related to the median age of a standard population. Hence there is no need to make use of the concept of remaining life expectancies. This allows for a direct explanation of falling lifetimes: The population age adjusted median life expectancy at birth may fall if the median of the age distribution at death falls.

Finally, it must be mentioned that computations of mean life expectancies should be avoided since they produce biased results. In our applications the median life expectancy at birth is always several years higher than the corresponding mean life expectancy. This is due to the skewness of the age distribution at death and its truncation at the age of 90 years.

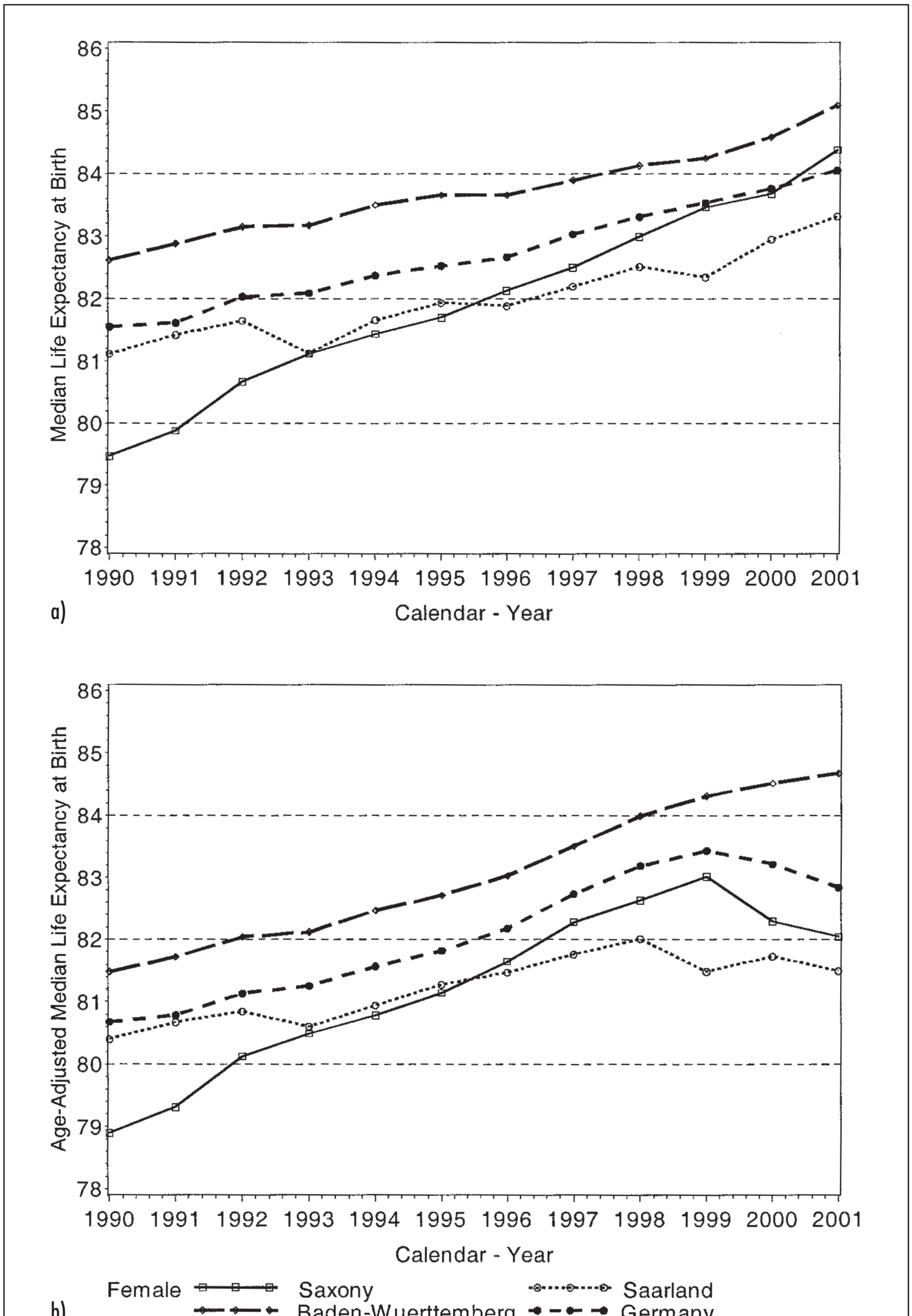

Fig. 4 Measures of populations' state of health: a) median life expectancy at birth, b) age-adjusted median life expectancy at birth

References

1. Halley E. An estimate on the decrease of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslaw. Philosophical Transactions, Royal Society, London, 1691.
2. Cutler SJ, Ederer F. Maximum utilization of the life table in analysing survival. Journal of Chronic Diseases 1958; 699-712.
3. Sanderson WC, Scherbov S. Average remaining lifetimes can increase as human populations age. Nature 2005; 435: 811-3.
4. Mehnert RF. Zur gesundheitlichen Entwicklung der saarländischen Bevölkerung im Vergleich zu Sachsen, Baden-Württemberg und Deutschland. Thesis of the Medical Faculty of the University of Saarland, Germany, 2004.

Correspondence to:
Prof. Dr. Uwe Feldmann
Institute of Medical Biometry, Epidemiology and Medical Informatics
University of Saarland
66421 Homburg/Saar, Germany
E-mail: uf@med-imbei.uni-saarland.de
www.uniklinik-saarland.de/imbei